

Lesson 6.3: Definite Integrals & The Fundamental Theorem of Calculus

Definite Integrals

A definite integral is written with upper and lower limits attached to an integration expression.

The value of a definite integral $\int_a^b f(x) dx$ may be thought of as _____ from the lower limit a (usually a left side boundary) to the upper limit b (usually a right side boundary), and between the curve of $f(x)$ and the x-axis.

The value of definite integrals may be _____, _____, or zero.

Unlike the previous integration process which introduced an indefinite integral representing a family of curves, a definite integral represents a **number value**.

Examples:

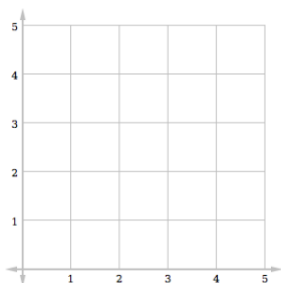
- Use your calculator to evaluate the following integrals.

a. $\int_{-3}^1 (x^3 - 6x) dx$

b. $\int_{-\sqrt{6}}^{\sqrt{6}} (x^3 - 6x) dx$

c. $\int_{-5}^5 |x^3 - 6x| dx$

- Use the idea of "signed area" to evaluate $\int_0^3 |2x - 1| dx$ without using a calculator.



- Set up a definite integral which could be used to find the area of the region bounded by the graph of $y = 2x^2 - 3x + 2$, the x-axis, and the vertical lines $x = 0$ and $x = 2$.

The Fundamental Theorem of Calculus

$$\text{If } f' \text{ is continuous on } [a, b], \text{ then } \int_a^b f'(x) dx = f(x) \Big|_a^b = f(b) - f(a)$$

Examples: Evaluate using the Fundamental Theorem of Calculus.

1. $\int_0^4 (2\sqrt{y} + 1) dy$

2. $\int_0^1 (4t + 1)^5 dt$

3. $\int_1^5 \frac{x}{\sqrt{2x-1}} dx$

4. $\int_0^{\frac{\pi}{2}} \cos(2x) dx$

Start Plus Accumulation Method (SPAM)

Since $\int_a^b f'(x) dx = f(b) - f(a)$, it follows that $f(b) = f(a) + \int_a^b f'(x) dx$. This means a function value at an endpoint can be found as a starting value plus a definite integral.

Examples:

1. If $f'(x) = 3x^2 + 3$ and $f(0) = 4$, find $f(2)$ without a calculator.

2. If an object's velocity is $v(t) = 2^t$ and $s(2) = 8$ find $s(3)$.