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Lesson 6.4: The Second Fundamental Theorem of Calculus

Warm-Up

1. $\int_0^x (2t-3)dt = \frac{2}{2}t^2 - 3t \Big|_0^x$
$= t^2 - 3t\Big|_0^x$
$= x^2 - 3x$

2. $\int_{10}^x f'(t)\,dt = f(x) - f(10)$

3. $\frac{d}{dx}\int_0^x (2t-3)\,dt = \frac{d}{dx}\left(t^2 - 3t\Big|_0^x\right)$
$= \frac{d}{dx}x^2 - 3x$
$= 2x - 3$

4. $\frac{d}{dx}\int_{10}^x f'(t)dt = f'(x)$

The Second Fundamental Theorem of Calculus

For any constant a, $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$
(if f is continuous from a to x)

Examples:

1. $\int_0^{x^2} f'(t)dt = f(x^2) - f(0)$

2. $\int_{x^3}^{2x} f'(t)dt = f(2x) - f(x^3)$

3. $\frac{d}{dx}\int_0^{x^2} f'(t)dt = f'(x^2)\cdot 2x$

4. $\frac{d}{dx}\int_{x^3}^{2x} f'(t)dt = 2f'(2x) - 3x^2 f'(x^3)$

The Second Fundamental Theorem (Chain Rule Version)

If u and v are functions of x, then:
$\frac{d}{dx}\int_u^v f(t)\,dt = f(v)v' - f(u)u'$

Examples: Find each of the following **without integrating.**

1. $\frac{d}{dx}\int_x^0 (2t-3)\,dt = (2(0)-3)(0)-(2x-3)(1)$
$= -2x+3$

2. $\frac{d}{dx}\int_{-1}^{x^3} (t^2+2t)\,dt = ((x^3)^2+2(x^3))(3x^2)-((-1)^2+2(-1))(0)$
$= (x^6+2x^3)(3x^2)$
$= 3x^8+6x^5$

3. $\frac{d}{dx}\int_{f(x)}^{g(x)} (2t-3)\,dt = ((2g(x)-3)\,g'(x)) - ((2f(x)-3)\,f'(x))$

4. $\frac{d}{dx}\int_2^5 (2t-3)\,dt = 0$

5. $\frac{d}{dx}\int_{2x}^{3x} (t^2+2t)\,dt = [((3x)^2+2(3x))(3)] - [((2x)^2+2(2x))(2)]$
$= [27x^2+18x]-[8x^2+8x]$
$= 19x^2+10x$

6. If $f(x)=\int_0^{3x^2}(1-t^2)^{10}dt$, then $f'(x) = 6x(1-9x^4)^{10}$
$\frac{d}{dx}\int_0^{3x^2}(1-t^2)^{10}dt = f'(x)$
$f'(x) = (1-(3x^2)^2)^{10}(6x) - 0$
$f'(x) = 6x(1-9x^4)^{10}$