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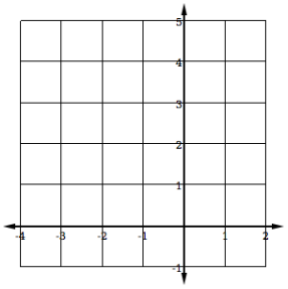
Lesson 6.6: Derivatives and Integrals of Exponential Functions

An exponential function is a function represented by a _____ with a variable _____ .

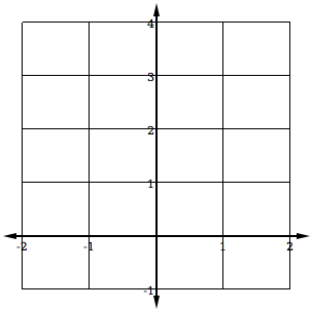
For example, $f(x) = 3^x$, $f(x) = e^x$, and $f(x) = 3^{x^2-5}$ are exponential functions.

Examples:

1. Use a calculator to carefully graph $y = 2^x$, $y = 5^x$, and $y = e^x$ in the same coordinate plane. Do you see any similarities in the graphs?



2. Using adjustments to the graph from Example 1, graph $f(x) = e^{-x} + 1$ without using a calculator. Write an equation for the graph's asymptote.



<u>Graphs of Exponential Functions</u>:

If $f(x) = a^x$ and $a > 1$, then	
1. The domain of $f(x)$ is: The range of $f(x)$ is:	2. The graph of $f(x)$ is
3. The *x*-axis is a	4. The y-intercept is: Another key point is:

Differentiating Exponential Functions

Examples: Differentiate.

1. $y = e^{x^2-3x}$ 2. $g(t) = e^{-\frac{3}{t}}$ 3. $f(v) = 3^{\sqrt{v}}$

4. Find the relative extrema of $f(x) = -xe^{-2x^2}$. List as points and do not use a calculator.

Integrating Exponential Functions

Examples: Compute the following integrals.

1. $\int_0^{\frac{1}{2}} 4^{x^2-1}x\,dx$ 2. $\int \frac{e^{\frac{2}{x}}}{3x^2}\,dx$ 3. $\int \frac{e^x-e^{-x}}{(e^x+e^{-x})^4}\,dx$ 4. $\int e^x \cos(e^x)\,dx$

Integration Involving the Natural Log Function

Differentiation and integration are inverse operations.

So if $\frac{d}{dx}\ln|x| = \frac{1}{x}$, then $\int \frac{1}{x} = \ln|x| + C$, and if $\frac{d}{dx}\ln|u| = \frac{u'}{u}$, then $\int \frac{u'}{u} = \ln|u| + C$

Log Rules

Note: Although it is true that both $\frac{d}{dx}\ln x = \frac{1}{x}$ and $\frac{d}{dx}\ln|x| = \frac{1}{x}$, $\int \frac{1}{x} = \ln|x| + C$ only.

Examples: Integrate.

1. $\int \frac{-3}{x}\, dx$
2. $\int \frac{P}{P^2+1}\, dP$
3. $\int \frac{9t^2-6t}{t^3-t^2}\, dt$
4. $\int \frac{\sec^2(x)}{\tan(x)}\, dx$

When integrating a fraction where the degree of the numerator $\geq$ the degree of the denominator, you will have to use long division (or creative thinking) to "split the fraction".
Examples: Integrate.

1. $\int \frac{x^2-4x+2}{x^2+2}\,dx$

2. $\int \frac{\ln(x)}{x}\,dx$

3. $\int \frac{1}{x(2-\ln(x))^3}\,dx$

4. $\int \frac{1}{\sqrt{x}-1}\,dx$

5. $\int \cot(x)\,dx$

6. $\int \tan(x)\,dx$

7. $\int \tan(2x)\,dx$

8. $\int \tan^2(x)dx$

Integral of Tangent and Cotangent
