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Lesson 7.1: Solving Differential Equations

Differential Equations

Differential Equations are equations with derivatives in them.

We will be working with differential equations in which you can separate variables.

You may be asked to find a general solution of the differential equation (which gives you a family of curves) or a particular solution (just one curve).

Procedure for Solving Differential Equations

1. Rewrite y' as $\frac{dy}{dx}$ (if necessary)
2. Multiply both sides of the equation by dx.
3. Separate variables (this is the most crucial step!)
4. Integrate both sides of the equation. * Remember to add c to one side!
5. Solve for y (if necessary).
6. Use an initial condition to solve for C. (if one is given)

*Note: Steps 5 & 6 are interchangeable

Examples:

1. Find the general solution of $x + 2yy' = 0$.

$dx\left(x + 2y\frac{dy}{dx}\right) = (0)dx$

$x\,dx + 2y\,dy = 0$

$-x\,dx \qquad -x\,dx$

$\int 2y\,dy = \int -x\,dx$

$\sqrt{y^2} = \sqrt{-\frac{1}{2}x^2 + C}$

$y = \pm\sqrt{-\frac{1}{2}x^2 + C}$

Write your solution to Example 1 as a pair of possible functions (in the form $y = f(x)$) for the particular solutions to the differential equation. Let $C = 5$

$y = \sqrt{-\frac{1}{2}x^2 + 5}$ *or* $y = -\sqrt{-\frac{1}{2}x^2 + 5}$

2. Find an equation of a function which contains the point $(0, -3)$, and whose slope is $\frac{xe^{x^2}}{y}$ for each point (x, y) on the curve.

$dx\left(\frac{dy}{dx}\right) = \left(\frac{xe^{x^2}}{y}\right)dx$

$y\,(dy) = \left(\frac{xe^{x^2}}{y}\,dx\right)y$

$\int y\,dy = \int xe^{x^2}dx$

$\frac{1}{2}y^2 = \frac{1}{2}e^{x^2} + C$

$\frac{1}{2}(-3)^2 = \frac{1}{2}e^{0^2} + C$

$\frac{1}{2}(9) = \frac{1}{2} + C$

$\frac{9}{2} = \frac{1}{2} + C$

$-\frac{1}{2} \qquad -\frac{1}{2}$

$C = 4$

$2\left(\frac{1}{2}y^2\right) = \left(\frac{1}{2}e^{x^2} + 4\right)2$

$\sqrt{y^2} = \sqrt{e^{x^2} + 8}$

$y = \pm\sqrt{e^{x^2} + 8}$

3. Find a general solution of $y - 2 = x\frac{dy}{dx}$.

$dx\,(y-2) = \left(x\frac{dy}{dx}\right)dx$

$(y-2)\,dx = x\,dy$

$\frac{1}{x}dx = \frac{1}{y-2}dy$

$\ln|x| = \ln|y-2| + C$

$e^{\ln|x|} = e^{(\ln|y-2| + C)}$

$|x| = e^{\ln|y-2|}e^{C}$

$|x| = C|y-2|$

$C|x| = |y-2|$

$C\sqrt{x^2} = \sqrt{(y-2)^2}$

$Cx = y - 2$

$\boxed{y = Cx + 2}$

Find a particular solution of $y - 2 = x\frac{dy}{dx}$ if $y(1) = \frac{1}{2}$.

$\frac{1}{2} = C(1) + 2$

$\frac{1}{2} = C + 2$

$-2 \qquad -2$

$-\frac{3}{2} = C$

$\boxed{y = -\frac{3}{2}x + 2}$