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Lesson 7.1: Trigonometric Identities

Fundamental Trig Identities

Reciprocal Identities		
$\csc(x) = \frac{1}{\sin(x)}$	$\sec(x) = \frac{1}{\cos(x)}$	$\cot(x) = \frac{1}{\tan(x)}$
$\tan(x) = \frac{\sin(x)}{\cos(x)}$	$\cot(x) = \frac{\cos(x)}{\sin(x)}$	
Pythagorean Identities		
$\sin^2(x)+\cos^2(x) = 1$	$\tan^2(x)+1 = \sec^2(x)$	$1+\cot^2(x) = \csc^2(x)$
Even-Odd Identities		
$\sin(-x) = -\sin(x)$	$\cos(-x) = \cos(x)$	$\tan(-x) = -\tan(x)$
Cofunction Identities		
$\sin(\frac{\pi}{2}-x) = \cos(x)$	$\tan(\frac{\pi}{2}-x) = \cot(x)$	$\sec(\frac{\pi}{2}-x) = \csc(x)$
$\cos(\frac{\pi}{2}-x) = \sin(x)$	$\cot(\frac{\pi}{2}-x) = \tan(x)$	$\csc(\frac{\pi}{2}-x) = \sec(x)$

Simplifying Trigonometric Expressions

We can use the above trig identities to help us simplify the same trigonometric expression in different ways.

Example: Simplify the expression $\cos(t) + \tan(t)\sin(t)$.

$$\cos(t)+\tan(t)\sin(t) = \cos(t) + \left(\frac{\sin(t)}{\cos(t)}\right)\sin(t)$$

$$= \frac{\cos^2(t)}{\cos(t)} + \frac{\sin^2(t)}{\cos(t)}$$

$$= \frac{\cos^2(t)+\sin^2(t)}{\cos(t)}$$ *Pythagorean Identity

$$= \frac{1}{\cos(t)}$$

$$= \sec(t)$$ * Reciprocal Identity

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Proving Trigonometric Identities

There are several other trigonometric identities that follow from the fundamental identities in the text box we created. As a result, we are able to prove new trig identities using the fundamental identities.

Guidelines for Proving Trigonometric Identities:

1. Choose one side of the equation to focus on.
2. Manipulate that side of the equation to look like the other side. Write any identities used.
3. State LHS = RHS ✓

Examples:

1. Verify algebraically that $\cos(\theta)\,(\sec(\theta) - \cos(\theta)) = \sin^2(\theta)$

$$LHS = \cos(\theta)(\sec(\theta) - \cos(\theta)) = \cos(\theta)\sec(\theta) - \cos^2(\theta)$$

$$= \cos(\theta)\frac{1}{\cos(\theta)} - \cos^2(\theta) \quad \text{*Reciprocal Identity}$$

$$= 1 - \cos^2(\theta)$$

$$= \sin^2(\theta) \quad \text{*pythagorean identity} \qquad \Rightarrow LHS = RHS ✓$$

$$= RHS$$

2. Verify algebraically that $2\tan(x)\sec(x) = \frac{1}{1-\sin(x)} - \frac{1}{1+\sin(x)}$

$$RHS = \frac{1}{1-\sin(x)} - \frac{1}{1+\sin(x)} = \frac{1+\sin(x)}{1-\sin^2(x)} - \frac{(1-\sin(x))}{1-\sin^2(x)}$$

$$= \frac{2\sin(x)}{1-\sin^2(x)} = \frac{2\sin(x)}{\cos^2(x)} \quad \text{*pythagorean identity}$$

$$= 2\,\frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\cos(x)} \quad \text{*Reciprocal Identity}$$

$$= 2\tan(x)\sec(x) = LHS \qquad \Rightarrow LHS = RHS ✓$$

3. Verify algebraically that $\frac{1+\cos(\theta)}{\cos(\theta)} = \frac{\tan^2(\theta)}{\sec(\theta)-1}$

$$LHS = \frac{1+\cos(\theta)}{\cos(\theta)} = \frac{1}{\cos(\theta)} + \frac{\cos(\theta)}{\cos(\theta)} = \sec(\theta) + 1 \quad \text{*reciprocal identity}$$

$$RHS = \frac{\tan^2(\theta)}{\sec(\theta)-1} \overset{\text{*pythag identity}}{=} \frac{\sec^2(\theta)-1}{\sec(\theta)-1} = \frac{(\sec(\theta)+1)\cancel{(\sec(\theta)-1)}}{\cancel{(\sec(\theta)-1)}} = \sec(\theta)+1$$

$$\Rightarrow LHS = RHS ✓$$

4. Substitute $\sin(\theta)$ for x in the expression $\sqrt{1-x^2}$ and simplify. Assume that $0 \le \theta \le \frac{\pi}{2}$.

Let $x = \sin(\theta)$

$$\sqrt{1-\sin^2(\theta)} = \sqrt{\cos^2(\theta)} \quad \text{*pythagorean identity}$$

$$= \cos(\theta)$$