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Lesson 7.2: Exponential Growth and Decay

Exponential Growth and Decay

Mathematical models in which the rate of change of a variable is proportional to the variable itself are common in both the business and scientific worlds.

Suppose that the rate of change of y (with respect to time) is proportional to y itself.

$$\boxed{\frac{dy}{dt} = k \cdot y}$$

Rate of change of y with respect to time	=	constant of proportionality	•	amount of substance y present at time t (y is a function of t)

1. Separate variables and solve the differential equation in the last notes box.

$$\frac{1}{y}dt\left(\frac{dy}{dt}\right) = (ky)\frac{1}{y}dt$$

$$\int \frac{1}{y}\,dy = \int k\,dt$$

$$e^{\ln|y|} = e^{kt + C}$$

$$|y| = e^{kt}e^{C}$$

$$|y| = Ce^{kt}$$

$$y = Ce^{kt}$$

Basic Law of Exponential Growth or Decay:

$$y = Ce^{kt}$$

Constants:

C is the initial value (the amount of the substance present at time $t=0$)

k is the constant of proportionality ($k>0 \to$ growth / $k<0 \to$ decay)

Variables:

t is the variable for time

y is the amount of substance present at time t. (y is a function of t)

2. What is the growth rate of the population of a city whose population triples every 100 years? Assume that the population growth can be modeled by the Basic Law of Exponential Growth, and express your answer as a percent (rounded to the nearest thousandth of a percent).

$$y = Ce^{kt}$$

$$\frac{3C}{C} = \frac{Ce^{100k}}{C}$$

$$3 = e^{100k}$$

$$\ln(3) = \ln(e^{100k})$$

$$\frac{\ln(3)}{100} = \frac{100k}{100}$$

$$k = \frac{\ln(3)}{100} \approx .010986 \approx 1.099\%$$

The growth rate is about 1.099%

3. Let y represent the mass, in pounds, of a radioactive element whose half-life is 4000 years. If there are 200 pounds of the element in an inactive mine, how much will still remain in 1000 years? Express your answer to 3 decimal place accuracy.

$$100 = 200e^{4000k}$$

$$\frac{1}{2} = e^{4000k}$$

$$\ln\left(\tfrac{1}{2}\right) = \ln(e^{4000k})$$

$$\frac{\ln\left(\frac{1}{2}\right)}{4000} = \frac{4000k}{4000}$$

$$k = \frac{\ln(\frac{1}{2})}{4000}$$

$$y = 200e^{\left(\frac{\ln(\frac{1}{2})}{4000}\right)(1{,}000)}$$

$$y \approx 168.179 \text{ lbs}$$

There will be about 168.179 pounds remaining after 1,000 years.