Name: ______________________ Period: ______

LESSON 7.2: Exponential Growth & Decay

WARM UP

Solve the equations below for x.

1. $4 = e^{2x}$

$\ln(4) = \ln(e^{2x})$

$\ln(4) = (2x)\ln(e)$

$\frac{\ln(4)}{2} = \frac{2x}{2}$

$x = \frac{1}{2}\ln(4)$

2. $27 = 3^{-x}$

$3^3 = 3^{-x}$

$\frac{3}{-1} = \frac{-x}{-1}$

$x = -3$

EXPONENTIAL growth AND decay

Today, we will be working with situations in which the ___rate___ of change of a variable is proportional to the variable itself:

$$\boxed{\frac{dy}{dt}} = \boxed{k}\boxed{y}$$

the rate of change of y with respect to time

constant of proportionality

amount of a substance present at time t (y is a function of t)

This model naturally occurs in the realms of both business and science.

Solving this ___differential___ equation gives us the formula for exponential growth and decay.

$$dt\left(\frac{dy}{dt}\right) = (ky)\,dt$$

$$\left(\frac{1}{y}\right)dy = (ky\,dt)\left(\frac{1}{y}\right)$$

$$\int \frac{1}{y}\,dy = \int k\,dt$$

$$e^{\ln|y|} = e^{kt+C}$$

$$|y| = e^{kt}\boxed{e^{C}}$$

just another constant...

$$|y| = Ce^{kt}$$

$$y = Ce^{kt}$$

The Basic Law of Exponential Growth or Decay

$$\boxed{y} = \boxed{C}e^{\boxed{k}\boxed{t}}$$

amount of substance present at time t

constant of proportionality (k<0 → decay / k>0 → growth)

time

initial amount of the substance present at t=0.

SOLVING PROBLEMS INVOLVING EXPONENTIAL *growth* **AND** *decay*	**Examples:**

1. What is the growth rate of the population of a city whose population doubles every 50 years? Assume that the population growth can be modeled by the Basic Law of Exponential Growth. Express your final answer as a percentage (rounded to the nearest thousandth of a percent).

$$y = Ce^{kt}$$

$$\frac{2C}{C} = \frac{Ce^{50k}}{C}$$

$$2 = e^{50k}$$

$$\ln(2) = \ln(e^{50k})$$

$$\ln(2) = (50k)\ln(e)$$

$$\frac{\ln(2)}{50} = \frac{50k}{50}$$

$$k = \frac{\ln(2)}{50}$$

$$k \approx 0.0138629$$

$$k \approx 1.386\%$$

The growth rate of the population is approximately 1.386%

2. Let P represent the mass (in pounds) of a radioactive element whose half-life is 3,000 years. If 400 pounds of the element was found in an inactive mine, how many pounds will remain in 3,200 years? Round your final answer with at least three decimal place accuracy.

$$P = Ce^{kt}$$

$$\frac{200}{400} = \frac{400e^{k(3000)}}{400}$$

$$\frac{1}{2} = e^{3000k}$$

$$\ln\left(\tfrac{1}{2}\right) = \ln(e^{3000k})$$

$$\ln\left(\tfrac{1}{2}\right) = (3000k)(\ln(e))$$

$$\frac{\ln(\frac{1}{2})}{3000} = \frac{3000k}{3000}$$

$$k = \frac{\ln(\frac{1}{2})}{3000}$$

$$P = 400e^{\left(\frac{\ln(1/2)}{3000}\right)t}$$

$$P(3200) = 400e^{\left(\frac{\ln(1/2)}{3000}\right)(3200)}$$

$$P(3200) \approx 190.968$$

After 3,200 years, there would be about 190.968 lbs of the element remaining.