# Lesson 7.4: Basic Trigonometric Equations

A trigonometric equation is an equation that contains trigonometric functions

For example, the trig identity: $\sin^2(\theta) + \cos^2(\theta) = 1$ is a trigonometric equation that is true for any value of $\theta$.

Solving Basic Trigonometric Equations

To solve a trigonometric equation, we need to find all values of $\theta$ that make the equation true. (Note: Solving and verifying (or proving) are not the same thing.)

Examples:

1. Solve the equation $\sin(\theta) = \frac{1}{2}$ for all values of $\theta$.

$\sin(\theta) = \frac{1}{2}$

any ∡ on the unit circle whose y-coordinate is $\frac{1}{2}$.

negative values too!

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \ldots$

$\theta = \frac{\pi}{6} + 2\pi k$ or $\theta = \frac{5\pi}{6} + 2\pi k$ where $k \in \mathbb{Z}$ integer

2. Solve the equation $\cos(\theta) = \frac{-\sqrt{2}}{2}$ for all values of $\theta$ and list 8 specific solutions.

$\cos(\theta) = -\frac{\sqrt{2}}{2}$

any ∡ on the unit circle whose x-coordinate is $-\frac{\sqrt{2}}{2}$

$\theta = \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}, \frac{19\pi}{4}, \frac{21\pi}{4}, -\frac{5\pi}{4}, -\frac{3\pi}{4}, \ldots$

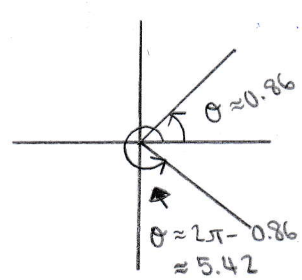
$\theta = \frac{3\pi}{4} + 2\pi k$ or $\theta = \frac{5\pi}{4} + 2\pi k$ where $k \in \mathbb{Z}$

3. Solve the equation $\cos(\theta) = 0.65$.

$\cos(\theta) = 0.65$

$\cos^{-1}(0.65) = \theta$

$\theta \approx 0.86$




$\theta \approx 0.86 + 2\pi k$ or $\theta \approx 5.42 + 2\pi k$ where $k \in \mathbb{Z}$

4. Find all solutions of the equations below.

a. $2\sin(\theta) - 1 = 0$

$$2\sin(\theta) - 1 = 0$$
$$\underline{\quad +1 \quad +1}$$
$$\frac{2\sin(\theta)}{2} = \frac{1}{2}$$
$$\sin(\theta) = \frac{1}{2}$$

$\theta = \frac{\pi}{6} + 2\pi k$ OR $\theta = \frac{5\pi}{6} + 2\pi k$ for $k \in \mathbb{Z}$

b. $\tan^2(\theta) - 3 = 0$

$$\tan^2(\theta) - 3 = 0$$
$$\underline{\quad +3 \quad +3}$$
$$\sqrt{\tan^2(\theta)} = \sqrt{3}$$
$$\tan(\theta) = \pm\sqrt{3}$$
$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \ldots$$

$\theta = \frac{\pi}{3} + \pi k$ OR $\theta = \frac{2\pi}{3} + \pi k$

where $k \in \mathbb{Z}$

Solving Trigonometric Equations by Factoring

Examples:

1. Solve the equation $2\cos^2(\theta) - 7\cos(\theta) + 3 = 0$

$$2\cos^2(\theta) - 7\cos(\theta) + 3 = 0$$

Let $\cos(\theta) = c$

$$2c^2 - 7c + 3 = 0$$
$$(2c - 1)(c - 3) = 0$$
$$2c - 1 = 0 \quad \text{OR} \quad c - 3 = 0$$

$2\cos(\theta) - 1 = 0$ OR $\cos(\theta) - 3 = 0$

$\underline{\quad +1 \quad +1}$ $\qquad$ $\underline{\quad +3 \quad +3}$

$\frac{2\cos(\theta)}{2} = \frac{1}{2}$ $\qquad$ $\cos(\theta) \neq 3$

$\cos(\theta) = \frac{1}{2}$

$\theta = \frac{\pi}{3} + 2\pi k$ OR $\theta = \frac{5\pi}{3} + 2\pi k$ for $k \in \mathbb{Z}$

2. Solve the equation $5\sin(\theta)\cos(\theta) + 4\cos(\theta) = 0$

$$5\sin(\theta)\cos(\theta) + 4\cos(\theta) = 0$$
$$\cos(\theta)(5\sin(\theta) + 4) = 0$$

$\cos(\theta) = 0.$ OR $5\sin(\theta) + 4 = 0$

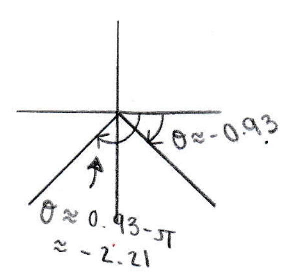
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$ $\qquad$ $\underline{\quad -4 \quad -4}$

$\frac{5\sin(\theta)}{5} = \frac{-4}{5}$

$\sin(\theta) = -\frac{4}{5}$

$\sin^{-1}(-\frac{4}{5}) = \theta$

$\theta \approx -0.93$




$\theta = \frac{\pi}{2} + \pi k$, $\theta \approx -0.93 + 2\pi k$, OR $\theta \approx -2.21 + 2\pi k$ where $k \in \mathbb{Z}$