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Lesson 7.4: Euler's Method

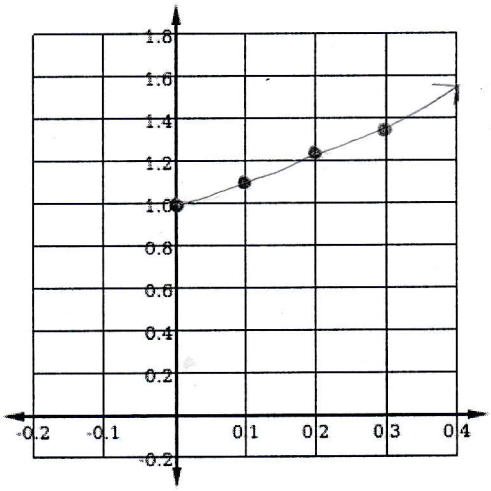
Euler's Method

Euler's Method is a more precise method of graphing an approximate solution to a differential equation.

Examples:

1. Use Euler's method to construct an approximate solution for the differential equation $\frac{dy}{dx} = y$. Start at the point $(0, 1)$ and use step size $\Delta x = 0.1$.

x	y	$\frac{dy}{dx}$	$\Delta y = (slope)\Delta x$
0	1	1	$\Delta y = (1)(0.1) = 0.1$
0.1	1.1	1.1	$\Delta y = (1.1)(0.1) = 0.11$
0.2	1.21	1.21	$\Delta y = (1.21)(0.1) = 0.121$
0.3	1.331		



$y(.3) \approx$ 1.331

2. Solve $\frac{dy}{dx} = y$ algebraically. Fill in the table with the actual values of y.

x	y
0	1
0.1	1.105
0.2	1.221
0.3	1.350

$\frac{1}{y}dx\left(\frac{dy}{dx}\right) = (y)\frac{1}{y}dx$

$\int \frac{1}{y}\,dy = \int 1\,dx$

$\ln|y| = x + C$

$e^{\ln|y|} = e^{x+C}$

$|y| = e^x e^C$

$|y| = C e^x$

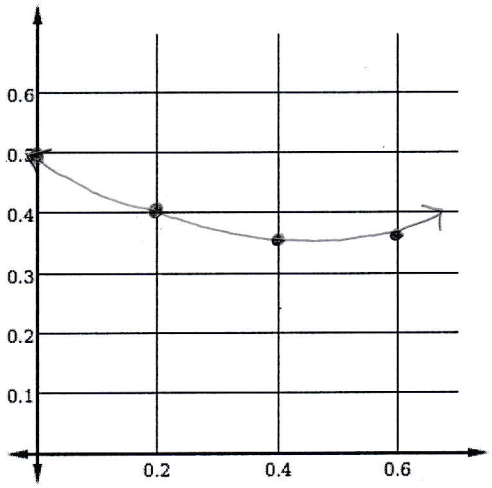
$1 = C e^0$

$1 = C$

$y = e^x$

3. Use Euler's Method to approximate the particular solution of the differential equation $y' = x - y$ passing through the point $(0, 0.5)$. Let $\Delta x = 0.2$ and do three steps $(n = 3)$. Graph the points.

x	y	$\frac{dy}{dx}$	$\Delta y = (slope)\Delta x$
0	0.5	-0.5	$\Delta y = (-0.5)(0.2) = -0.1$
0.2	0.4	-0.2	$\Delta y = (-0.2)(0.2) = -0.04$
0.4	0.36	0.04	$\Delta y = (0.04)(0.2) = 0.008$
0.6	0.368		



$y(.6) \approx$ 0.368

4. Sketch a particular solution to the differential equation $y' = x - y$ passing through the point $(0, 0.5)$ using the slope field given. Do the two graphs coincide?

Yes, the graphs in example 3 & 4 coincide. The slope of both curves at each point line up and the points are about the same on both curves.