Entry #: 10

Lesson 7.5: More Trigonometric Equations

Using Trig Identities to Solve Trigonometric Equations

Examples:

1. Solve the equation $1 + \sin(\theta) = \cos^2(\theta)$

$1+\sin(\theta) = \cos^2(\theta)$

$1+\sin(\theta) = 1-\sin^2(\theta)$ (Pythagorean Identity)

$-1 \quad -1$

$\sin(\theta) = -\sin^2(\theta)$

$\sin^2(\theta) + \sin(\theta) = 0$

$\sin(\theta)(\sin(\theta)+1) = 0$

$\sin(\theta) = 0 \quad \sin(\theta)+1 = 0$

$\theta = 0, \pi, \cdots$ $\quad -1 \quad -1$

$\sin(\theta) = -1$

$\theta = \frac{3\pi}{2}, \frac{7\pi}{2}, \cdots$

$\theta = \pi k$ OR $\theta = \frac{3\pi}{2} + 2\pi k$ $k \in \mathbb{Z}$

2. Solve the equation $\sin(2\theta) - \cos(\theta) = 0$

$\sin(2\theta) - \cos(\theta) = 0$

$2\sin(\theta)\cos(\theta) - \cos(\theta) = 0$ (double ∠ identity)

$\cos(\theta)(2\sin(\theta) - 1) = 0$

$\cos(\theta) = 0 \quad 2\sin(\theta) - 1 = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \cdots$ $\quad +1 \quad +1$

$\frac{2\sin(\theta)}{2} = \frac{1}{2}$

$\sin(\theta) = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \cdots$

$\theta = \frac{\pi}{2} + \pi k$, $\theta = \frac{\pi}{6} + 2\pi k$, OR $\theta = \frac{5\pi}{6} + 2\pi k$

$k \in \mathbb{Z}$

3. Solve the equation $\cos(\theta) + 1 = \sin(\theta)$ on the interval $[0, 2\pi)$. ← not infinite solutions

$(\cos(\theta)+1)^2 = (\sin(\theta))^2$

$\cos^2(\theta) + 2\cos(\theta) + 1 = \sin^2(\theta)$

$\cos^2(\theta) + 2\cos(\theta) + 1 = 1 - \cos^2(\theta)$

$2\cos^2(\theta) + 2\cos(\theta) = 0$

$2\cos(\theta)(\cos(\theta)+1) = 0$

$2\cos(\theta) = 0 \quad \cos(\theta)+1 = 0$

$\cos(\theta) = 0 \quad \cos(\theta) = -1$

$\theta = \frac{\pi}{2}$ & $\frac{3\pi}{2}$ $\quad \theta = \pi$

$\theta = \frac{\pi}{2}, \pi,$ & $\frac{3\pi}{2}$

Solving Trigonometric Functions with Multiples of Angles

Examples:

1. Find the solutions on the interval $[0,2\pi)$ for the equation $2\sin(3\theta) - 1 = 0$. Then, state all possible solutions.

$2\sin(3\theta) - 1 = 0$

$+1 \quad +1$

$\frac{2\sin(3\theta)}{2} = \frac{1}{2}$

$\sin(3\theta) = \frac{1}{2}$

$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$

$3\theta = \frac{\pi}{6} + 2\pi k \qquad 3\theta = \frac{5\pi}{6} + 2\pi k$

$\theta = \frac{\pi}{18} + \frac{2k\pi}{3}$ OR $\theta = \frac{5\pi}{18} + \frac{2k\pi}{3}$ $\quad k \in \mathbb{Z}$

2. Find all solutions of the equation $\sqrt{3}\tan\left(\frac{\theta}{2}\right) - 1 = 0$.

$\sqrt{3}\tan\left(\frac{\theta}{2}\right) - 1 = 0$

$+1 \quad +1$

$\frac{\sqrt{3}\tan\left(\frac{\theta}{2}\right)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\tan\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{3}}$

$\frac{\theta}{2} = \frac{\pi}{6}, \frac{7\pi}{6}$

$\theta = \frac{\pi}{3}, \frac{7\pi}{3}$

$\frac{\theta}{2} = \frac{\pi}{6} + \pi k$

$\theta = \frac{\pi}{3} + 2\pi k \quad k \in \mathbb{Z}$