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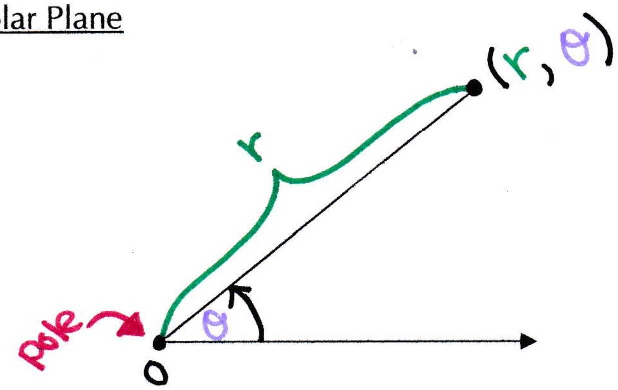
Lesson 8.1: Polar Coordinates

What are Polar Coordinates?

The **polar coordinate system** uses distances (radius) and direction (angle → θ) to specify the location of point on a plane.

Setting Up a Polar Plane

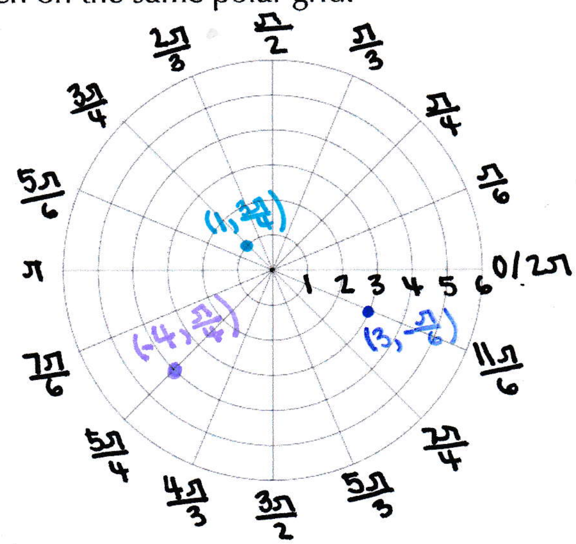


Examples: Plot the points whose polar coordinates are given on the same polar grid.

1. $\boxed{\left(1, \frac{3\pi}{4}\right)}$

2. $\boxed{\left(3, -\frac{\pi}{6}\right)}$

3. $\boxed{\left(-4, \frac{\pi}{4}\right)}$



The Relationship Between Polar and Rectangular Coordinates

1) To change from polar → rectangular:

$$x = r\cos(\theta) \quad \& \quad y = r\sin(\theta)$$

2) To change from rectangular → polar:

$$r^2 = x^2 + y^2 \quad \& \quad \tan(\theta) = \frac{y}{x} \; (x \neq 0)$$

Examples: Convert the following coordinates from polar to rectangular or vice versa.

1. $\left(4, \frac{2\pi}{3}\right)$

$x = 4\cos\left(\frac{2\pi}{3}\right)$

$x = 4\left(-\frac{1}{2}\right)$

$x = -2$

$y = 4\sin\left(\frac{2\pi}{3}\right)$

$y = 4\left(\frac{\sqrt{3}}{2}\right)$

$y = 2\sqrt{3}$

$(-2, 2\sqrt{3})$

2. $(2, -2)$

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$(2)^2 + (-2)^2 = r^2$

$4 + 4 = r^2$

$\sqrt{8} = \sqrt{r^2}$

$r = \pm 2\sqrt{2}$

$\tan(\theta) = -\frac{2}{2}$

$\tan(\theta) = -1$

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}, \ldots$

$\left(2\sqrt{2}, \frac{7\pi}{4}\right)$ OR $\left(-2\sqrt{2}, \frac{3\pi}{4}\right)$

Examples: Convert the following equations from polar to rectangular or vice versa.

1. $x^2 = 4y$

$(r\cos(\theta))^2 = 4r\sin(\theta)$

$\frac{r^2\cos^2(\theta)}{r\cos^2(\theta)} = \frac{4r\sin(\theta)}{r\cos^2(\theta)}$

$r = \frac{4\sin(\theta)}{\cos^2(\theta)}$

2. $\frac{r}{\sec(\theta)} = \frac{5\sec(\theta)}{\sec(\theta)}$

$r\cos(\theta) = 5$

$x = 5$

3. $r = 2 + 2\cos(\theta)$

$r^2 = 2r + 2r\cos(\theta)$

$x^2 + y^2 = 2r + 2x$

$-2x \qquad -2x$

$(x^2 + y^2 - 2x)^2 = (2r)^2$

$(x^2 - 2x + y^2)^2 = 4r^2$

$(x^2 - 2x + y^2)^2 = 4(x^2 + y^2)$