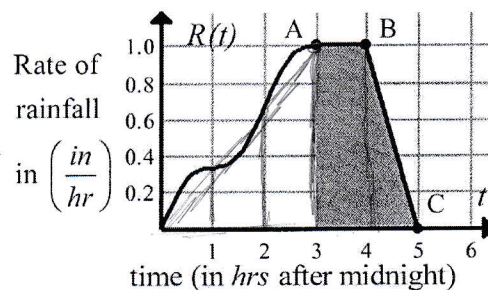


Lesson 8.1: Rate Graphs & Average Value of a Function

Interpretation of "Rate Graphs"

Examples: The graph at right models the rate of rainfall in inches per hour from midnight until 6:00am during a tropical rainstorm.



- Write a complete sentence to explain what Point A on the graph represents. Include the units in your answer.

Point A represents: 3 hours after midnight, rain was falling at a rate of 1 inch per hour.

- What is the slope of the graph between Point A and Point B?

$$m(A, B) = 0$$

- Write a complete sentence to explain the meaning of your answer in #2. Include numbers and units in your answer.

This means the rate of rainfall remains constant (does not change) between 3:00AM and 4:00AM.

- What is the slope of the graph between Points B and C?

$$m(B, C) = -1$$

- Write a complete sentence to explain the meaning of your answer in #4. Include numbers and units in your answer.

This means that the rate of rainfall is decreasing by 1 inch per hour.

- Find $\int_3^5 R(t) dt$.

$$\int_3^5 R(t) dt \approx 1(1) + \frac{1}{2}(1)(1) = 1 + \frac{1}{2} = \frac{3}{2}$$

- Write a complete sentence to explain the meaning of your answer in #6. Include numbers and units in your answer.

From 3:00AM to 5:00AM there was 1.5 inches of rain.

- Approximate the value of $\int_0^6 R(t) dt$ using geometric regions. Show computations.

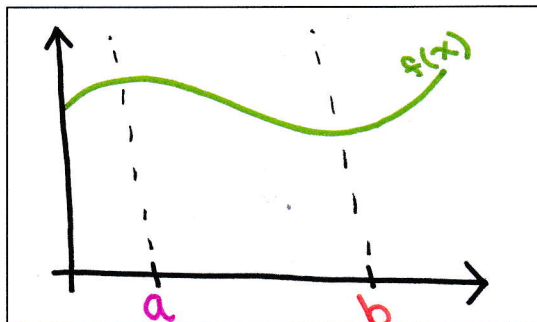
$$\int_0^6 R(t) dt = \int_0^3 R(t) dt + \int_3^5 R(t) dt + \int_5^6 R(t) dt \approx 1.5 + 1.5 + 0 = 3$$

- Write a complete sentence to explain the meaning of your answer in #8. Include numbers and units in your answer.

From 12:00AM to 6:00AM, there was about 3 inches of rain.

Average Value of a Function

The **average value** of a function represents its average "height".



The Average Value of $f(x)$ on $[a, b]$:

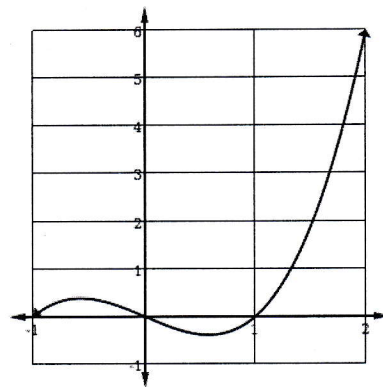
$$f(c) = f_{\text{avg}} = \frac{\int_a^b f(x) dx}{b-a} = \frac{1}{b-a} \int_a^b f(x) dx$$

* f must be continuous on $[a, b]$

Examples:

1. Find the average value of $f(x) = x^3 - x$ on the interval $[0, 2]$ without using a calculator.

$$\begin{aligned} f_{\text{avg}} &= \frac{1}{2-0} \int_0^2 x^3 - x dx \\ &= \frac{1}{2} \left[\left(\frac{1}{4}x^4 - \frac{1}{2}x^2 \right) \Big|_0^2 \right] \\ &= \frac{1}{2} \left(\frac{1}{4}(2)^4 - \frac{1}{2}(2)^2 \right) \\ &= \frac{1}{2} (4 - 2) \\ &= \frac{1}{2} (2) \\ &= 1 \end{aligned}$$



2. Use your calculator to find the value of c in the interval $[0, 2]$ where $f(c) =$ the average value you found in #1.

$$\begin{aligned} x^3 - x &= 1 \\ x^3 - x - 1 &= 0 \Rightarrow x = 1.325 \end{aligned}$$