Entry #: 39

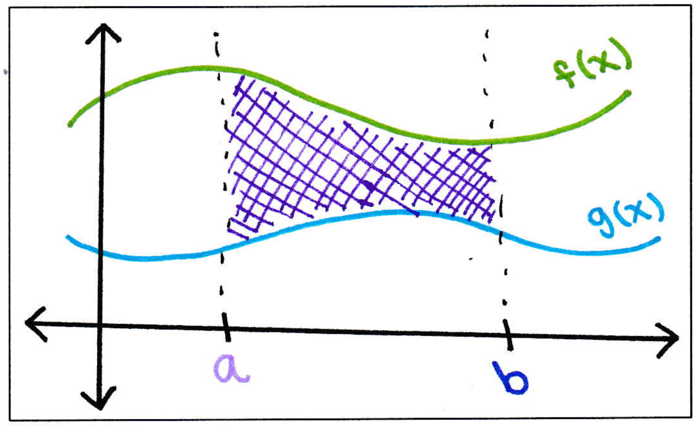
Lesson 8.2: Area Between Curves

$\int_a^b f(x)\,dx$ produces a value "signed area" which may be positive, negative, or zero.

However, if you are asked to find an actual area , that area can't be negative .

Area of a Region Between Two Curves:

$$A = \int_a^b f(x) - g(x)\,dx$$



For functions of x, $A = \int_a^b (top\ curve - bottom\ curve)\,dx$.

For functions of y, $A = \int_a^b (right\ curve - left\ curve)\,dy$.

Example:

Find the area of the region bounded by $y = x^2 + 2$, $y = x$, $x = 0$, and $x = 1$.

$$\int_0^1 (x^2+2) - x\,dx = \int_0^1 x^2 - x + 2\,dx$$
$$= \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x \Big|_0^1$$
$$= \frac{1}{3}(1)^3 - \frac{1}{2}(1)^2 + 2(1)$$
$$= \frac{1}{3} - \frac{1}{2} + 2$$
$$= \frac{2}{6} - \frac{3}{6} + \frac{12}{6}$$
$$= \frac{11}{6}$$

Sometimes, you have to find where two curves intersect to determine "boundaries" for your region(s). These intersections will provide you with limits of integration for your integral(s). You must show an equation set up, even when using a calculator to find intersections.

Examples:

1. Find the area of the region bounded by $x = y^2 - 3$ and $y = x + 1$.

$y = x + 1 \Rightarrow x = y - 1$

$y - 1 = y^2 - 3$

$y^2 - y - 2 = 0$

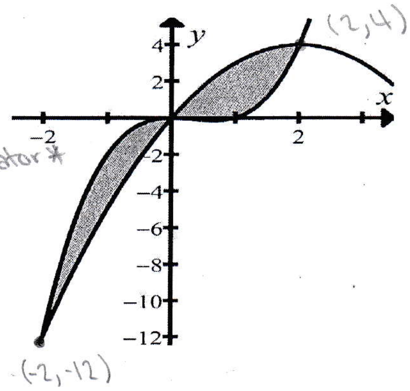
$(y+1)(y-2) = 0$

$y = -1$ OR $y = 2$

$$\int_{-1}^{2} (y-1)-(y^2-3)\,dy$$
$$= \int_{-1}^{2} -y^2 + y + 2\,dy$$
$$= -\tfrac{1}{3}y^3 + \tfrac{1}{2}y^2 + 2y \Big|_{-1}^{2}$$
$$= \left[-\tfrac{1}{3}(2)^3 + \tfrac{1}{2}(2)^2 + 2(2)\right] - \left[-\tfrac{1}{3}(-1)^3 + \tfrac{1}{2}(-1)^2 + 2(-1)\right]$$
$$= \left[-\tfrac{8}{3} + \tfrac{6}{3} + \tfrac{12}{3}\right] - \left[\tfrac{2}{6} + \tfrac{3}{6} - \tfrac{12}{6}\right]$$
$$= \tfrac{10}{3} + \tfrac{7}{6} = \tfrac{27}{6} = \tfrac{9}{2}$$

2. Find the total area of the regions located between the two curves as shown. You may use a calculator.

$f(x) = x^3 - x^2$ and $g(x) = -x^2 + 4.1x$



$$A = \int_{-2}^{0} f(x)-g(x)\,dx + \int_{0}^{2} g(x)-f(x)\,dx$$
$$= \int_{-2}^{0} x^3 - 4.1x\,dx + \int_{0}^{2} -x^3 + 4.1x\,dx$$

calculator

$$= 8.4$$

OR $A = \int_{-2}^{2} |g(x) - f(x)|\,dx = 8.4$

3. Without using a calculator set up an integral for the area of one region bounded between the graphs of $y = \sin(\theta)$ and $y = \cos(\theta)$.

$$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin(\theta) - \cos(\theta)\,d\theta = -\cos(\theta) - \sin(\theta)\Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$
$$= \left[-\cos\left(\tfrac{5\pi}{4}\right) - \sin\left(\tfrac{5\pi}{4}\right)\right] - \left[-\cos\left(\tfrac{\pi}{4}\right) - \sin\left(\tfrac{\pi}{4}\right)\right]$$
$$= \left[\tfrac{\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2}\right] - \left[-\tfrac{\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2}\right]$$
$$= \frac{2\sqrt{2}}{2} + \frac{2\sqrt{2}}{2}$$
$$= \sqrt{2} + \sqrt{2}$$
$$= 2\sqrt{2}$$