

## Lesson 8.3: Volume of Solids with Known Cross Sections

### Commonly Used Area Formulas (For Cross Sections)

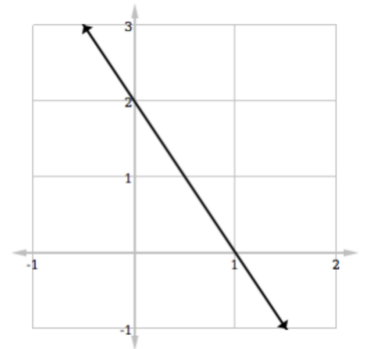
Square	Rectangle	Semicircle	Triangle	Equilateral Triangle

### Volumes of Solids with Known Cross Sections

For cross sections perpendicular to the _____ :
For cross sections perpendicular to the _____ :

### Examples:

- Find the volume of the solid whose base is a triangle bounded by  $y = -2x + 2$ ,  $x = 0$ , and  $y = 0$ , and whose cross sections are squares which are perpendicular to the x-axis.



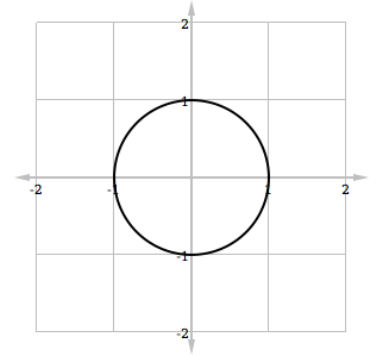
- Set up (but do not integrate) integrals for the volumes of the solids with the same base area as in Example 1, but whose cross sections are semicircles perpendicular to the x-axis.

- Set up (but do not integrate) an integral for the volume of a solid whose base is bounded by  $y = -x^2 + 2$  and  $y = x$  and whose cross sections are rectangles of height  $\frac{1}{4}$  perpendicular to the x-axis.

4. Set up integrals for the volumes of the solids whose base is the circle  $1 = x^2 + y^2$  and whose cross sections are:

a. Equilateral triangles perpendicular to the y-axis.

b. Rectangles whose heights are three times their bases and whose bases are perpendicular to the y-axis.

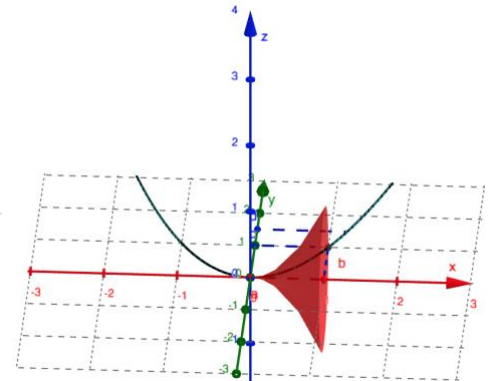
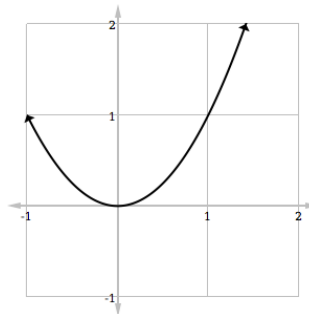


Circular cross sections can be formed by revolving very thin (essentially no width) rectangles about an \_\_\_\_\_ of revolution.

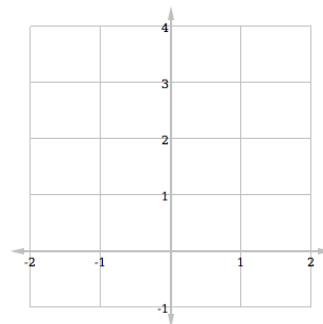
These circular cross sections are more commonly known as \_\_\_\_\_ .

Disc Formula

5. Set up an integral for the volume of the solid formed by revolving the region bounded by  $y = x^2$ ,  $y = 0$ , and  $x = 1$  about the x-axis.



6. Find the volume of the solid formed by revolving the region in Quadrant I bounded by  $y = x^2$ ,  $x = 0$ , and  $y = 4$  about the  $y$ -axis.

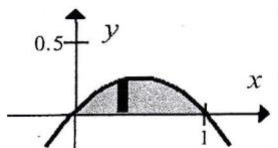


When a region is revolved around a line which is not one of its boundaries, its volume is formed from the sum of volumes of \_\_\_\_\_ (at least some of the discs have holes in them).

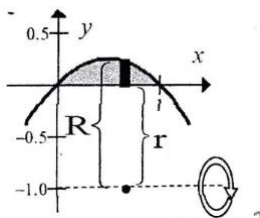
Washer Formula

7. Set up integrals for the volumes of the solids formed by revolving the region bounded by  $y = -x^2 + x$  and  $y = 0$ .

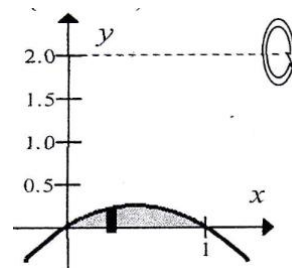
a. About the  $x$ -axis (discs)



b. About  $y = -1$  (washers)

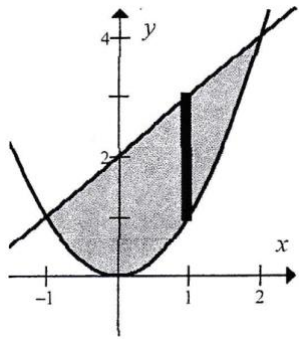


c. About  $y = 2$  (washers)



8. Set up integrals for the volumes of the solids formed by revolving the region bounded by  $y = x^2$  and  $y = x + 2$

a. About the x-axis.



b. About the line  $y = 4$ .

