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Lesson 8.4: Shell Method Volume and Arc Length

Shell Method

The key to using the shell method, is that the representative element (rectangle) must be _______________________ to the axis of revolution.

Recall that for discs or washers, the element had to be _______________________ to the axis of revolution.

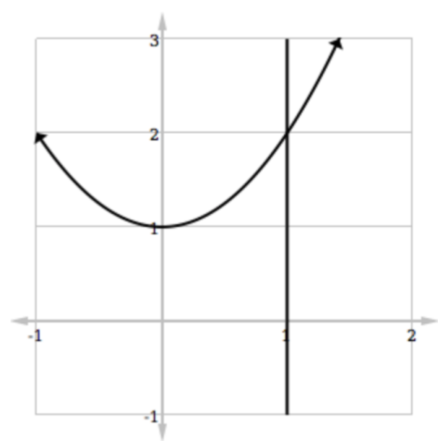
Memory Device:

Revolving rectangular elements about a parallel axis produces ______________________ shells (like wrappings around a toilet paper roll).

The Volume Formula for Shell Method:

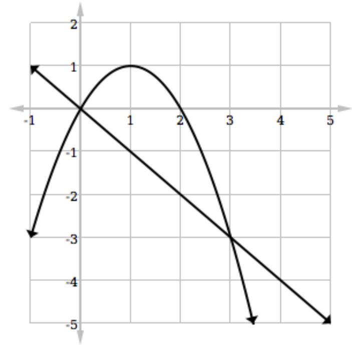
Examples:

1. Find the volume of the solid formed by revolving the region bounded by $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis.

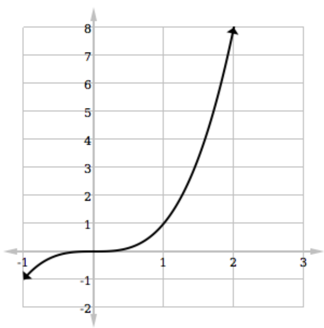


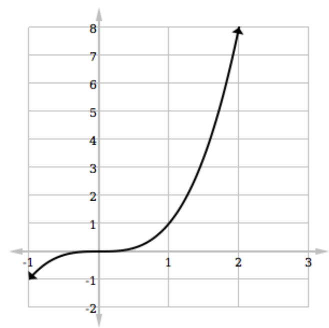
Why would using the disc method for this problem be much more difficult?

2. Set up (but do not integrate) an integral giving the volume of the solid formed by revolving the region bounded by $y = 2x - x^2$ and $y = -x$ about the y-axis.



3. Use both the shell method and the disc method to find the volume formed by revolving the region bounded by $y = x^3$, $x = 2$, and $y = 0$ about the x-axis.





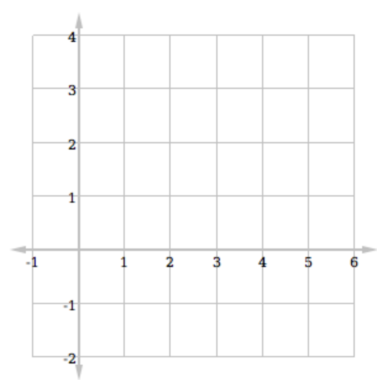
For revolutions about lines other than the x-axis and y-axis, the formula is still:

$$V = 2\pi \int_a^b rh\, dx\ (or\ dy)$$

However, r is slightly harder to find. (Remember both r and h must be nonnegative.)

Examples:

4. Use the shell method to set up integrals which could be used to find the volumes of the solids formed when the region bounded by $y = \sqrt{x-1}$, $y = 0$, and $x = 5$ is revolved about:

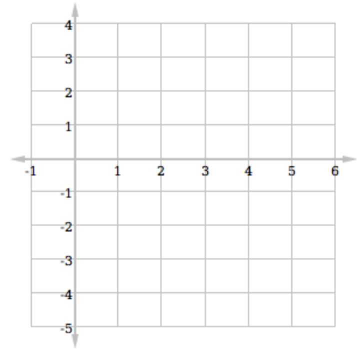


   a. The y-axis

   b. $x = -2$

   c. $x = 5$

5. Find the volume of the solid formed by revolving the region bounded by $x = y^2$ and $x = 4$ about the line $y = -3$.



6. Set up an integral (but do not integrate) which could be used to find the volume of the solid which would be formed if the region from Example 5 were revolved about the line $y = 3$.

Arc Length

Example:

7. Find the length of the arc of the curve $y = 3x^3 - 3x + 2$ on the interval $[0,2]$.