Name: _____ Period: _____

Sequences & Series Mixed Practice

1. What is the difference between an arithmetic and a geometric sequence?
   In an arithmetic sequence, we add a constant to each term to get the next, where in a geometric sequence, we multiply a constant to each term to get the next.

2. Find the next three terms of each sequence

   a. 9, 16, 23, 30, 37, 44 (+7 +7)

   b. 100, -200, 400, -800, 1600, -3200 (×-2 ×-2)

   c. -8, -5, -2, 1, 4, 7 (+3 +3)

3. Find the first three terms of each sequence where d is the common difference, and r is the common ratio

   a. $a_1 = 576, r = -\frac{1}{2}$ 576, -288, 144

   b. $a_1 = 2, d = 13$ 2, 15, 17

   c. $a_1 = \frac{5}{8}, d = \frac{3}{8}$ $\frac{5}{8}$, 1 $\left(\frac{8}{8}=1\right)$, $\frac{11}{8}$

4. Find a_8 if $a_n = 4 + 3n$.

   $a_8 = 4 + 3(8)$
   $= 4 + 24$
   $= 28$

5. Find a_7 if $a_n = 12\left(\frac{1}{2}\right)^{n-1}$.

   $a_7 = 12\left(\frac{1}{2}\right)^{7-1} = 12\left(\frac{1}{2}\right)^6 = 12\left(\frac{1}{64}\right) = \frac{12}{64} = \frac{3}{16}$

6. Find a_{12} for -17, -13, -9, … (+4 +4)

   $a_n = -17 + (n-1)(4)$
   $a_{12} = -17 + (12-1)(4) = -17 + (11)(4) = -17 + 44 = 27$

7. Find a_8 for 4, -12, 36, … (×-3 ×-3)

   $a_n = (4)(-3)^{n-1}$
   $a_8 = (4)(-3)^{8-1}$
   $a_8 = 4(-3)^7$
   $a_8 = -8748$

8. Find a_{14} if $a_1 = 3$ and the common difference is $d = 7$

$a_n = 3 + (n-1)(7)$

$a_{14} = 3 + (14-1)(7) = 3 + (13)(7) = 94$

9. Find a_8 if the common ratio $r = 3$ and $a_1 = \frac{1}{3}$

$a_n = \frac{1}{3}(3)^{n-1}$

$a_8 = \frac{1}{3}(3)^{8-1} = \frac{1}{3}(3)^7 = 3^6 = 729$

10. Write the equation for the *n*th term for each sequence

a. 7, 16, 25, 34, … (+9 +9 +9) $a_n = 7 + (n-1)(9)$

$a_n = 9n - 2$

b. 36, 12, 4, … (×⅓ ×⅓) $a_n = 36\left(\frac{1}{3}\right)^{n-1}$

11. Find the 10th term of the arithmetic sequence given the following information: $a_3 = 55, a_7 = 115$

$55 = a_1 + (3-1)d$

$55 = a_1 + 2d$

$55 = a_1 + 2(15)$

$55 = a_1 = 30$

$a_1 = 25$

$115 = a_1 + (7-1)d$

$115 = a_1 + 6d$

$-\ 55 = -a_1 - 2d$

$\frac{60}{4} = \frac{4d}{4}$

$d = 15$

$a_n = 25 + (n-1)(15)$

$a_{10} = 25 + (10-1)(15)$

$a_{10} = 25 + (9)(15)$

$a_{10} = 160$

12. Find the 7th term of the geometric sequence given the following information: $a_1 = 9, a_5 = 144$

$144 = 9(r)^{5-1}$

$\frac{144}{9} = \frac{9r^4}{9}$

$\sqrt[4]{r^4} = \sqrt[4]{16}$ $r = 2$

$a_n = 9(2)^{n-1}$

$a_7 = 9(2)^{7-1}$

$a_7 = 9(2)^6$

$a_7 = 576$

Write the first 5 terms of each sequence described below with recursive equations:

13. $a_1 = -3$, $a_{n+1} = 3a_n + 10$

$a_2 = 3(-3) + 10 = 1$

$a_3 = 3(1) + 10 = 13$

$a_4 = 3(13) + 10 = 49$

$a_5 = 3(49) + 10 = 157$

$-3, 1, 13, 49, 157, \ldots$

14. $a_1 = 5$, $a_n = a_{n-1} - 4$

$a_2 = 5 - 4 = 1$

$a_3 = 1 - 4 = -3$

$a_4 = -3 - 4 = -7$

$a_5 = -7 - 4 = -11$

$5, 1, -3, -7, -11, \ldots$

15. $a_1 = 2$, $a_2 = -1$, $a_{n+2} = a_{n+1} + 4a_n$

$a_3 = -1 + 4(2) = 7$

$a_4 = 7 + 4(-1) = 3$

$a_5 = 3 + 4(7) = 31$

$2, -1, 7, 3, 31, \ldots$

Series Questions

Find the value of each series below.

16. $\sum_{j=1}^{3}(2j-6)$ $= (2(1)-6)+(2(2)-6)+(2(3)-6)$
$= (-4)+(-2)+(0)$
$= -6$

17. $\sum_{k=0}^{3} 2(4)^k$ $= 2(4)^0 + 2(4)^1 + 2(4)^2 + 2(4)^3$
$= 2 + 8 + 32 + 128$
$= 170$

18. $6\sum_{j=1}^{4} j^j$ $= 6\left[(1)^1 + (2)^2 + (3)^3 + (4)^4\right]$
$= 6\left[1 + 4 + 27 + 256\right]$
$= 6(288)$
$= 1728$

19. Find $2\sum_{k=3}^{6} x_k$ if $x_3 = 2$, $x_4 = -4$, $x_5 = 8$ and $x_6 = 10$

$2\sum_{k=3}^{6} x_k = 2\left[2 + (-4) + (8) + 10\right]$
$= 2(16)$
$= 32$

20. Write this series using sigma notation: $2 + 9 + 16$

(+7, +7)

$\sum_{n=1}^{3} 7n-5$

$a_n = 2 + (n-1)(7)$
$a_n = 7n - 5$

21. Write this series using sigma notation: $3 + 12 + 48 + 192 + 768$

(×4, ×4, ×4, ×4)

$\sum_{n=1}^{5} 3(4)^{n-1}$

$a_n = 3(4)^{n-1}$

Evaluate each arithmetic series.

22. $a_1 = -2, \quad a_8 = 33, \quad S_8 = ?$

$S_8 = 8\left(\frac{-2+33}{2}\right) = 8\left(\frac{31}{2}\right) = 124$

23. $\sum_{n=1}^{50}(7n-1)$ $= 50\left(\frac{6+349}{2}\right)$

$= 8,875$

$a_1 = 7(1)-1$

$= 6$

$a_{50} = 7(50)-1$

$= 350-1$

$= 349$

Evaluate each geometric series.

24. $\sum_{n=1}^{11} 5 \cdot 3^{n-1}$ $= 5\left(\frac{1-3^{11}}{1-3}\right) = 442{,}865$

25. $\underbrace{1, 2, 4, 8, \ldots}_{\times 2 \ \times 2 \ \times 2}$ $\quad n = 25 \quad$ $\sum_{n=1}^{25} 2^{n-1} = 1\left(\frac{1-2^{25}}{1-2}\right) = 33{,}554{,}431$

$a_n = 1(2)^{n-1}$

26. Determine the number of terms n in the following geometric series.

$a_1 = 2, \quad r = 5, \quad S_n = 62$

$62 = 2\left(\frac{1-5^n}{1-5}\right)$

$(-4)(31) = \left(\frac{1-5^n}{-4}\right)(-4)$

$\frac{-124}{-1} = \frac{1-5^n}{-1}$

$\frac{-125}{-1} = \frac{-5^n}{-1}$

$125 = 5^n$

$5^3 = 5^n$

$n = 3$

* 3 terms