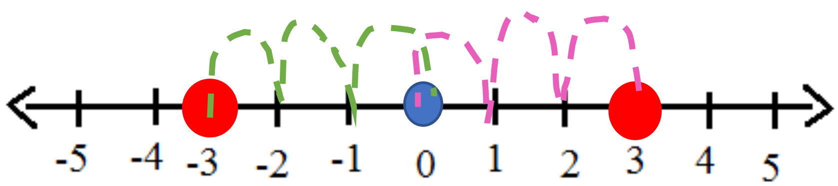
Solving Absolute Value Inequalities

The absolute value is a number's distance from zero. Since distance is always a positive number, the absolute value of any number is always positive.

For example:

$|-3| = 3$ and $|3| = 3$

This is because -3 and 3 are both 3 units away from zero, as shown below:



We use this idea when solving absolute value inequalities.

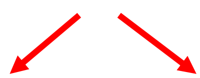
Since we can use two different inputs to get the same output in our absolute value function, most of these absolute value inequalities will have **two boundary points**.

How to Solve Absolute Value Inequalities

Example: $7 \leq |\, 2x + 1|$

Step 1: Solve for the boundary points. To do this, treat the inequality as if it were an equality.

$$7 = |\, 2x + 1\, |$$



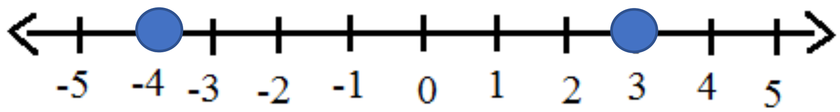
$2x + 1 = 7$ $\quad$ $2x + 1 = -7$

$2x = 6$ $\quad$ $2x = -8$

$x = 3$ $\quad$ $x = -4$

So, my two boundary points are 3 and -4.

Step 2: Plot both boundary points on a number line. Remember, if it is $\leq or \geq$ we use a closed dot and if it is $< or >$ it is an open dot .



Step 3: Use test points to determine where you need to shade your graph. Since there are three regions that can be shaded, we need **3 test points**. Plug your test points into your original inequality. If the test point makes the inequality true, **shade that region**.

$$7 \leq |2x + 1|$$

Test: $x = 0$

$7 \leq |2(0) + 1|$

$7 \leq |1|$

$7 \leq 1$ ***False!***

Test: $x = -5$

$7 \leq |2(-5) + 1|$

$7 \leq |-10 + 1|$

$7 \leq |-9|$

$7 \leq 9$ ***True!***

Test: $x = 4$

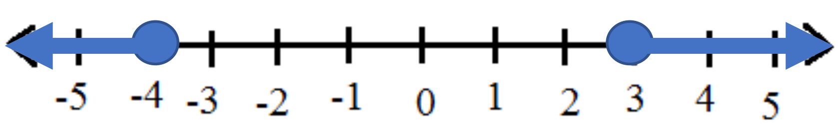
$7 \leq |2(4) + 1|$

$7 \leq |8 + 1|$

$7 \leq |9|$

$7 \leq 9$ ***True!***

Step 4: Shade your graph based on your test points.



Step 5: Write your final solution in terms of x. Note: Steps 4 and 5 would be considered a final solution for this problem.

$$x \leq -4 \quad or \quad x \geq 3$$