Name: _____ Period: _____

UNIT 4: SEQUENCES & SERIES REVIEW

Directions: Read each questions carefully. Show all your work.

1. Find the first five terms and the 100th term for each of the sequences below.

a. $a_n = 2n + 2$

$a_1 = 2(1) + 2 = 4$
$a_2 = 2(2) + 2 = 6$
$a_3 = 2(3) + 2 = 8$
$a_4 = 2(4) + 2 = 10$
$a_5 = 2(5) + 2 = 12$
$a_{100} = 2(100) + 2 = 202$

b. $a_n = \frac{2}{n}$

$a_1 = 2$
$a_2 = 1$
$a_3 = \frac{2}{3}$
$a_4 = \frac{1}{2}$
$a_5 = \frac{2}{5}$
$a_{100} = \frac{1}{50}$

c. $a_n = n^2 - 2$

$a_1 = (1)^2 - 2 = -1$
$a_2 = (2)^2 - 2 = 2$
$a_3 = (3)^2 - 2 = 7$
$a_4 = (4)^2 - 2 = 14$
$a_5 = (5)^2 - 2 = 23$
$a_{100} = (100)^2 - 2 = 9{,}998$

d. $a_n = \frac{\sqrt{n}}{n+1}$

$a_1 = \frac{\sqrt{1}}{1+1} = \frac{1}{2}$
$a_2 = \frac{\sqrt{2}}{2+1} = \frac{\sqrt{2}}{3}$
$a_3 = \frac{\sqrt{3}}{3+1} = \frac{\sqrt{3}}{4}$
$a_4 = \frac{\sqrt{4}}{4+1} = \frac{2}{5}$
$a_5 = \frac{\sqrt{5}}{5+1} = \frac{\sqrt{5}}{6}$
$a_{100} = \frac{\sqrt{100}}{100+1} = \frac{10}{101}$

2. Find the nth term for the sequences given below.

a. $\frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$

$a_n = \frac{1}{(n+1)^2}$

b. 7,5,3,1, …

$a_n = 7 - 2(n-1)$
$a_n = -2n + 9$

c. 3,6,12,24,48, …

$a_n = 3(2)^n$

d. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

$a_n = \frac{n}{n+1}$

e. 2,5,14,41, …

$a_n = 3a_{n-1} - 1$

*recursive equation required

f. $1, \sqrt{2}, \sqrt{3}, 2, \ldots$

$a_n = \sqrt{n}$

3. Find the first five terms of the sequences below that are defined recursively.

a. $a_1 = 3$ and $a_n = 2a_{n-1} - 2$

$a_1 = 3$
$a_2 = 2(3) - 2 = 4$
$a_3 = 2(4) - 2 = 6$
$a_4 = 2(6) - 2 = 10$
$a_5 = 2(10) - 2 = 18$

b. $a_1 = 1$, $a_2 = 3$ and $a_n = a_{n-1} + a_{n-2}$

$a_1 = 1$
$a_2 = 3$
$a_3 = 1 + 3 = 4$
$a_4 = 3 + 4 = 7$
$a_5 = 4 + 7 = 11$

4. Write a **recursive** rule for the sequences given below.

a. 1,1,2,3,5,8, ...

$a_n = a_{n-1} + a_{n-2}$
$* a_1 = 1 \; \& \; a_2 = 1$

b. 2,5,11,23, ...

$a_n = 2a_{n-1} + 1$
$* a_1 = 2$

c. $1,5,4,-1,-5, \ldots$

$a_n = a_{n-1} - a_{n-2}$
$* a_1 = 1 \; \& \; a_2 = 5$

d. $10,5,\frac{5}{2},\frac{5}{4}, \ldots$

$a_n = \frac{a_{n-1}}{2}$
$* a_1 = 10$

5. Evaluate the sums below. Simplify your final answer.

a. $\sum_{k=1}^{4} k^2 = (1)^2 + (2)^2 + (3)^2 + (4)^2$
$= 1 + 4 + 9 + 16$
$= 30$

b. $\sum_{k=2}^{6} (k-3) = (2-3) + (3-3) + (4-3) + (5-3) + (6-3)$
$= -1 + 0 + 1 + 2 + 3$
$= 5$

c. $\sum_{k=1}^{4} \frac{1}{k} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$
$= \frac{12}{12} + \frac{6}{12} + \frac{4}{12} + \frac{3}{12}$
$= \frac{25}{12}$

d. $\sum_{k=2}^{4} \frac{k}{k-1} = \frac{2}{2-1} + \frac{3}{3-1} + \frac{4}{4-1}$
$= 2 + \frac{3}{2} + \frac{4}{3}$
$= \frac{12}{6} + \frac{9}{6} + \frac{8}{6}$
$= \frac{29}{6}$

6. Are the sequences below arithmetic, geometric or neither? If it is arithmetic, find the common difference. If it is geometric, find the common ratio.

a. $1, 2, 4, 8, 16, \ldots$

Geometric

$r = 2$

b. $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10} \ldots$

Neither

c. $0, 1, 4, 9, 16, \ldots$

Neither

d. $5, 3, 1, -1, -3, \ldots$

Arithmetic

$d = -2$

e. $1, 2, 3, 4, 5, 6, \ldots$

Arithmetic

$d = 1$

f. $-\frac{1}{2}, -\frac{1}{4}, -\frac{1}{8}, -\frac{1}{16}, \ldots$

Geometric

$r = \frac{1}{2}$

g. $1, 2, 6, 24, 120, \ldots$

Neither

h. $\frac{2}{3}, -\frac{2}{9}, \frac{2}{27}, -\frac{2}{81}, \ldots$

Geometric

$r = -\frac{1}{3}$

7. Find the indicated partial sum in the problems below.

a. Find the sum of the first 54 even numbers (starting at 2).

$2, 4, 6, 8, \cdots \quad a_n = 2n \quad a_{54} = 108$

$S_{54} = 54\left(\frac{2+108}{2}\right)$

$S_{54} = 2{,}970$

b. Find the sum of the first 49 natural numbers.

$1, 2, 3, 4, \ldots \quad a_n = n \quad a_{49} = 49$

$S_{49} = 49\left(\frac{1+49}{2}\right)$

$S_{49} = 1{,}225$

c. Find S_8 for the sequence: $\frac{2}{3}, -\frac{2}{9}, \frac{2}{27}, -\frac{2}{81}, \ldots$

$a_n = \frac{2}{3}\left(\frac{-1}{3}\right)^{n-1}$

$S_8 = \frac{2}{3}\left(\frac{1-(-1/3)^8}{1-(-1/3)}\right)$

$= \frac{3280}{6561}$

≈ 0.500

d. Find S_{10} for the sequence: $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$

$a_n = 2\left(\frac{1}{2}\right)^{n-1}$

$S_{10} = 2\left(\frac{1-(1/2)^{10}}{1-(1/2)}\right)$

$= \frac{1023}{256}$

≈ 3.996

8. Do the geometric series below converge or diverge? Explain. If it converges, find its value.

a. $\frac{6}{7}+\left(\frac{6}{7}\right)^2+\left(\frac{6}{7}\right)^3+\left(\frac{6}{7}\right)^4+\cdots = 6$

$r=\frac{6}{7}$ $\left|\frac{6}{7}\right|<1$

$\Rightarrow$ This geometric series converges

$\sum_{n=1}^{\infty}\left(\frac{6}{7}\right)^n = \frac{6/7}{1-6/7} = \frac{6/7}{1/7} = \frac{6}{7}\cdot\frac{7}{1} = 6$

b. $2-1+\frac{1}{2}-\frac{1}{4}+\cdots = \sum_{n=1}^{\infty}2\left(-\frac{1}{2}\right)^{n-1} = \frac{4}{3}$

$r=-\frac{1}{2}$ $\left|-\frac{1}{2}\right|<1$

$\Rightarrow$ This geometric series converges.

$\sum_{n=1}^{\infty}2\left(-\frac{1}{2}\right)^{n-1} = \frac{2}{1-(-1/2)} = \frac{2}{3/2} = 2\cdot\frac{2}{3} = \frac{4}{3}$

c. $\frac{6}{5}+\left(\frac{6}{5}\right)^2+\left(\frac{6}{5}\right)^3+\left(\frac{6}{5}\right)^4+\cdots = \sum_{n=1}^{\infty}\left(\frac{6}{5}\right)^n$

$r=\frac{6}{5}$ $\left|\frac{6}{5}\right|>1$

$\Rightarrow$ This geometric series diverges.

d. $1-2+4-8+\cdots = \sum_{n=1}^{\infty}(-2)^{n-1}$

$r=-2$ $|-2|>1$

$\Rightarrow$ This geometric series diverges.

9. Expand the binomials below.

a. $(x+2)^3 = \binom{3}{0}x^3(2)^0+\binom{3}{1}x^2(2)^1+\binom{3}{2}x^1(2)^2+\binom{3}{3}x^0(2)^3$

$= x^3+6x^2+12x+8$

b. $(a+b)^4 = \binom{4}{0}a^4b^0+\binom{4}{1}a^3b^1+\binom{4}{2}a^2b^2+\binom{4}{3}a^1b^3+\binom{4}{4}a^0b^4$

$= a^4+4a^3b+6a^2b^2+4ab^3+b^4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

c. $(x-5)^4 = \binom{4}{0}x^4(-5)^0+\binom{4}{1}x^3(-5)^1+\binom{4}{2}x^2(-5)^2+\binom{4}{3}x^1(-5)^3+\binom{4}{4}x^0(-5)^4$

$= x^4-20x^3+150x^2-500x+625$

d. $(a-2b)^3$

$= \binom{3}{0}a^3(-2b)^0+\binom{3}{1}a^2(-2b)^1+\binom{3}{2}a^1(-2b)^2+\binom{3}{3}a^0(-2b)^3$

$= a^3-6a^2b+12ab^2-8b^3$